Solutions

1 ***Method :*** *Find out all possible prime numbers.*

The sum of the width and length (together they form the "semiperimeter") is 30. We need to find 2 prime numbers to represent the width and length with sum 30.

Width (unit)	Length (unit)	Area (unit^2)
7	23	161
11	19	209
13	17	221

Hence, the greatest area the rectangle could have is 221 square units.

Solutions

2 ***Method 1 :*** *Analyse the possibility of picking 3 same colour balls.*

If Angie picks 5 balls, there could be three balls of the same colour among them. However, it is possible that she picked 2 balls of red, 2 balls of blue and 1 ball of white. She cannot be sure that she has three balls of the same colour.

If Angie picks 6 balls, it is possible that she picked 2 balls of each colour. Again, she cannot be sure that she has three balls of the same colour.

If Angie picks a 7 ball, it must result in having 3 balls of the same colour.

Hence, the least number of balls Angie must pick to ensure that there are 3 balls of same colour is 7 balls.

Method 2 : *Consider the worst scenario.*

In the worst scenario, Angie will pick different colour ball every time.

Assume that she picks :

Time	Colour of ball
1 st	Red
2 nd	White
3 rd	Blue
4 th	Red
5 th	White
6 th	Blue
7 th	Red / White / Blue

For the 7 th time, no matter what colour of the ball she picks, she will have 3 balls of the same colour.

Solutions

3 ***Method 1 :*** *Simplified the factorial.*

Notice that

$9! = 9 \times 8 \times 7 \times 6!$

$7! = 7 \times 6!$

$3! \times 5! = 3 \times 2 \times 1 \times 5!$

$= 6 \times 5!$

$= 6!$

We have $\dfrac{9!-7!}{3!\times 5!} = \dfrac{9\times 8\times 7\times 6! - 7\times 6!}{6!}$

$= 9 \times 8 \times 7 - 7$

$= \mathbf{497}$

In general, we have $n! = n \times (n-1)!$. For example, $6! = 6 \times 5!$.

Method 2 : *Calculate step by step and expend the factorial.*

Notice that

$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$3! \times 5! = 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

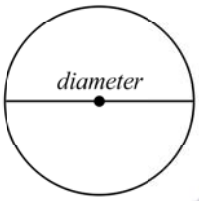
We have $\dfrac{9!-7!}{3!\times 5!} = \dfrac{9\times 8\times 7\times \cancel{6\times 5\times 4\times 3\times 2\times 1} - 7\times \cancel{6\times 5\times 4\times 3\times 2\times 1}}{\cancel{6\times 5\times 4\times 3\times 2\times 1}}$

$= 9 \times 8 \times 7 - 7$

$= \mathbf{497}$

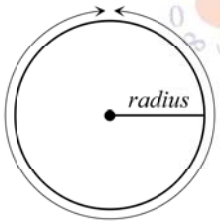
Solutions

4



A diameter of a circle is 2 times of its radius.

$$\text{Radius} = \frac{\text{diameter}}{2} = \frac{28}{2} = 14 \text{ cm}$$



The circumference of a circle is the radius of the circle times 2π.

$$\begin{aligned}\text{circumference} &= \text{radius} \times 2\pi \\ &= 14 \times 2\pi \\ &= 14 \times 2 \times \frac{22}{7} \\ &= 88 \text{ cm}\end{aligned}$$

Method 1 : *Divide the distance with the circumference.*

$$\text{Number of turns make} = \frac{\text{distance travelled}}{\text{circumference of the wheel}}$$

$$= \frac{1.1\text{km}}{88\text{ cm}}$$

$$= \frac{1100\text{ m}}{0.88\text{ m}}$$

$$= 1250 \text{ complete turns}$$

The wheel makes 1250 complete turns.

Method 2 : *Draw a table to calculate the number of complete turns.*

1 turn = circumference of the circle
= distance travel
= 88 cm

Number of complete turns	Distance travelled
1	1 × 0.88 = 0.88 m
2	2 × 0.88 = 1.76 m
3	3 × 0.88 = 2.64 m
...	...
1250	**1250 × 0.88 = 1100 m**

Solutions

5 ***Method 1 :*** *Make assumption.*

To simplify our calculation, let's assume Kenny caught 10 rats both in day 1 and day 2. Total rats caught by Kenny is 20 rats.
Then, Sammy caught 8 rats in day 1.

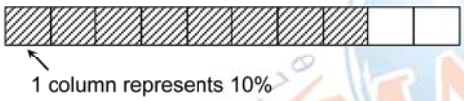
In order to make even, Sammy needs to catch at least 12 rats in the second day, which means Sammy has to catch 4 more rats.

That is $\frac{4}{8} \times 100\% =$ **50% more.**

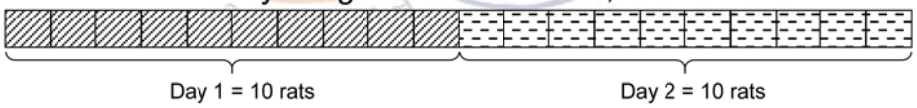
Method 2 : *Draw simple chart.*

Day 1:
Assume that Kenny caught 10 rats.

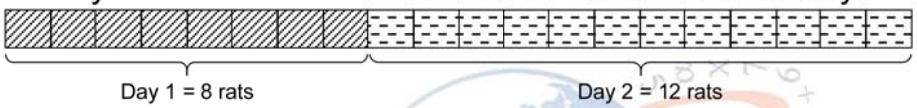
Sammy caught 20% less than Kenny, it means Sammy caught 80% = 8 rats.



Day 2:
Assume that Kenny caught another 10 rats, total is 20 rats.



Sammy need to catch 12 rats in order to be even with Kenny.



In the second day, Sammy has to catch (12 – 8 =) 4 more rats compare to Day 1.

$$\frac{\text{Number of additional rats in Day 2}}{\text{Number rats in Day 1}} = \frac{4}{8} \times 100\% = \textbf{50\% more}$$